Scientific Knowledge and Solving Problems Modelling, Representation and Processing

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Abstract

This article presents several important topics that show the importance of knowledge and solving problems in the development of scientific knowledge. At present the scientific and technical development, solving problems in a different field (math, science, physics, chemistry etc.) is a creative activity, by building a reasoning, generation, describing the following activities: demonstration process (deduction and reasoning) to show the existence of a solution or several solutions and / or to determine the exact effective solutions; computational process (algorithm) to codify a demonstration, a method or technique to solve in order to determine (possibly approximate) exact solutions. In the problem-solving processes require demonstrative thinking, a algorithms thinking. From the methodological point of view, we need to recast usual problems explicitly and properly resolve their mathematical. If the computer should use to develop algorithmic methods. In both cases you must know the limits of thinking demonstration. You should also know the limits of performance computing and algorithmic thinking. Every science is based on the theories, theorems (laws) and hypotheses that have been identified, studied and demonstrated by the strengthening, development and evolution in time of sciences. The article presents the problem of Gauss and Green theorem used to calculate the area of any polygon. Finally, we propose the following meta-model: problem solution = modelling + processing; modelling = knowledge + representation; processing = language + interpretation.

Keywords: Mathematical Models, Algorithmically Method, Computer Graphics, Gauss's Problem, Green's Theorem

1 Introduction and Literature

Today, Computer Science (Informatics) is among exact sciences along with mathematics, physics and chemistry. If in 70 years at the university level, there were several disciplines own science, today there are complex areas of computer science: Programming and Software Engineering, Computer Networks and Computing, Databases and information systems, programming and Web development, computer graphics and reality virtual, computational geometry, modelling and simulation, parallel and distributed Computing, artificial intelligence and expert systems, knowledge engineering (Vlada, 2005; Vlada and Țugui, 2006). A major requirement of today's knowledge society makes the educational systems of countries to be in a high dynamic. Change theories, disciplines, specializations, skills, and even in the sciences are constantly changing.

Today, one can say with certainty that the Mathematics and Computer Science are scientists who have contributed to a rapidly developing *Information and Communication Technologies* (ICT), in addition to other sciences and areas: Automation, Electronics, Electrical Engineering, Telecommunications etc. Information technology is the technology required for processing, in particular electronic computers use to convert, process and transmit information. Therefore, the computer is only device that theoretical concepts are implemented. Professor Edsger Dijkstra said: "*In Informatics you have to do with the computer, as you with the telescope in astronomy*." Informatics expression is a word that comes from the word alignment Information and Mathematics.

Computer Science history proceeding the time of occurrence digital computer. Before 1920, the term "*computer*" referred to a person who performed calculations (an official). The first researchers in what was to be called Computer Science, such as Kurt Gödel, Alonzo Church and Alan Turing, were interested in the computational problem. Computer Science (Informatics) is characterized by the most spectacular evolutions of the impact on human activity. Computer (Computer System) includes technologies of which man has never dreamt. Although at the beginning the use of computer was regarded with reservation, nowadays most of the people are convinced by the performance and utility of computer in all activities.

At present the scientific and technical development, solving problems in a different field (math, science, physics, chemistry etc.) is a creative activity, by building a reasoning, generation, describing the following activities: demonstration process (deduction and reasoning) and computational process (algorithm). Today, the specialists working in a certain field face different complex problems, many of these requiring the use of computer and software products.

2 Mathematics and Computer Science

Mathematics is the oldest of the exact sciences and Informatics has emerged and developed as a science in the second half of the 20th century (after 1960, when already emerged modern computer - designed for Hungarian mathematician John von Neumann (1903-1957) and develop theories, methods and techniques of data processing / information), being the newest. And today's report is recognized on John von Neumann's EDVAC Report 1945 (von Neumann, 1945)**,** EDVAC (*Electronic Discrete Variable Automatic Computer*) is one of the first electronic computers that utilized the binary system that first began performing basic tasks in 1951 (von Neumann, access 2009).

World in those years for pioneering the field of IT and computer use, and Romania has made an important contribution by the school of logic and data created by Romanian mathematician Grigore C. Moisil (1906-1973). Professor Gr. C. Moisil had outstanding contributions to the development of Informatics in Romania and in the formation of the first generation of informaticians. He had a contribution to the introduction and use of the first electronic computing machine in our country. Particularly valuable are the contributions made by Grigore C. Moisil the algebraic theory of automatic mechanisms. He developed new methods of analysis and synthesis of finite automatic and structural theory of them. He entered algebras called him lukasiewicziene trivalent and polyvalent (known today-*Moisil Lukasiewicz algebras*) and it has used in the study of logic and circuit switching (Vlada, 2005; Vlada and Țugui, 2006). For these contributions, post-mortem in 1996, Gr C. Moisil (O'Connor and Robertson, 2009) received the Computer Pioneer Award of IEEE (received award for his work "*For polyvalent logic switching circuits*."). The example provided by Moisil was followed by generations of mathematicians and informaticians contributions that have internationally recognized, both in scientific research and the use of computers for the overall development of the Romanian society and international.

2.1 Mathematics and scientific method

The word "mathematics" comes from the Greek **μάθημα (máthema)** which means "science, knowledge or learning"; **μαθηματικός (mathematikós)** means "*one who likes learning*". The terms "model", "hypothesis", "theory" and "theorem" has other meanings in science than in the usual language. Scientists use the term "model" to express the description of something, specifically something that can be used to make predictions that can be tested by experiment or observation. In the modern sense, mathematics is the investigation of structures defined in an abstract axiomatic using formal logic. Investigate the structures of mathematics often have their roots in natural sciences, often in physics. Mathematics and investigates and defines the structure and its own theories, in particular to synthesize and unify multiple fields in mathematical theory, single, a method that facilitates generally generic methods of calculation. Mathematics is generally defined as the science which studies the patterns of structure, operations in time and space.

The most important function of mathematics in science is the role that is the expression of scientific models. Processes of observation and grouping the results of experiments, creating assumptions and predictions often require **mathematical models**. Branches of mathematics most often used in science include the calculation and statistics, although almost every branch of mathematics has applications, even areas "pure" such as number theory and topology. Mathematical models are based on **scientific methods**. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, graphs, theoretic models etc.

These and other types of models can overlap, with a given model involving a variety of abstract structures. Mathematical models are used not only in the natural sciences and engineering disciplines (such as physics, chemistry, biology, earth science, meteorology, and engineering) but also in the social sciences (such as economics, psychology, sociology and political science); physicists, engineers, computer scientists, and economists use mathematical models most extensively.

Grigore C. Moisil say "*All what is correct thinking is either mathematics or feasible to be transposed in a mathematical model*".

"*Sciences are models and virtual representations of knowledge*" (Vlada, 2008a).

2.2 Computer Science and solving problems

Competence and experience in solving problems using computer can be permanent only if it is the dependency **System Computer-Algorithmic-Programming**, and if efforts are undertaken to acquire new knowledge and knowledge of all relevant aspects of the physical and virtual model. The entire research and development of software in the field of Information Technology is determined by the invention, design, development, testing, and implementation of useful algorithms and performance.

Wide variety of algorithms and their great applicability in all fields, makes the subject is always present and in a continuous change and improvement. In essence, solving a problem is expressed by the encoding of universe problem and the reasoning for demonstration (Vlada, 2005; Vlada and Ţugui, 2006). Stages of solving a problem with the computer: *the problem, mathematical model, algorithm, program, computer, results and verification solutions* [Fig. 1].

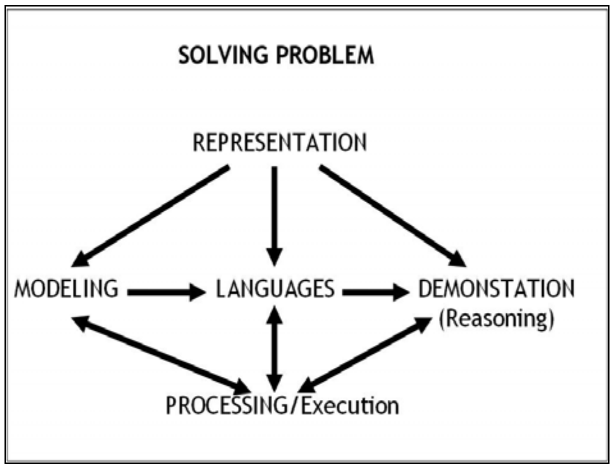

Fig. 1 The evolution modelling in solving problems

The performance of IT developer is determined by experience and expertise gained in conducting the two stages (ANALYSIS, PROGRAMMING):

- *object thinking stage* (ANALYSIS / Projection) - method of analysis and description of the problem by defining the correct objects, the types of objects, relationships between objects and specific operators (UAP development, stage design and analysis-design);
- *algorithmic thinking stage* (PROGRAMMING / execution) - the choice and proper application methods of solving the exact specification of the operators of processing objects, the correct representation of algorithmic strategies, codified representation of objects and processing according to a programming language (and algorithm development program; stage programming - coding implementation and enforcement).

„*Natural environments are ruled by languages. Computer Science use artificial languages. Languages exist therefore, not for communication purposes alone, but particularly for knowledge.*" (Vlada, 2005)

Practice solving problems using computer (programming languages or specialized software) resulted in time, various approaches based PC performance. It also depends on the methods and techniques on implementing advanced solution. Addressing a theoretical problem solving does not guarantee its practice with the computer, and vice versa. We illustrate this in the next section.

Language of knowledge (Vlada and Sarah Nica, 2009**)**:

- *Natural languages* (the languages of the peoples) - "*entity*"=word; lexical constructions describe states, images, actions, etc.
- *Languages of sciences* (used in the fields of science: mathematics, physics, chemistry, economics, etc.) - "*entity*"= knowledge/meet; study of objects and relationships between objects in the fields of mathematics, physics, chemistry, computer science, biology, economics, etc.
- *Artificial languages* (used computer) consisting of
  - Procedural Programming languages - "*entity*" =memory location
  - Functional Programming Languages - "*entity*" = item list
  - Logic Programming Languages - "*entity* "= object, clause
  - Object Oriented Programming - "*entity*" = object
  - Web Programming Languages - "*entity*" = web/multimedia elements
  - Languages for Databases - "*entity*" = registration
  - Languages for Computer graphics - "*entity*" = graphics object
  - Languages for Modeling-Simulation - "*entity* "= event
  - Languages for Operating Systems- "*entity*" = process/ task
  - Languages for Artificial Intelligence - "*entity*" = object/ knowledge

Definition. A language of knowledge is virtual system/logical

L = (V, Sin, Sem, O, C, T, Tc),

where

V = vocabulary / alphabet, Sin = syntax (rules), Sem = semantics (rules), O = objects, C = concepts / terms, T = theories / methods / techniques to solve, Tc = treasury of knowledge (knowledge base). (Vlada, 2005)

Using computers in language led to the conclusion that they are effectively used for processing, not only for communication. Develop programs to solve problems with a computer led to the development and evolution of all sciences. In most countries, research programs, development and innovation are the number of increasingly large and the results are not expected leave. Meanwhile, continuous improvement, knowledge and use of new knowledge in the field of activity should be the major goal of each specialist. They demonstrate that the necessary knowledge and experience to achieve consistent results on various topics of research and development. Also bring important arguments concerning the modelling, representation and processing, all contributing to the performance of technologies.

3 Gauss’s problem solved by a computer

Karl Friedrich GAUSS (177-1855) is world’s most famous mathematicians. The German mathematician Karl Friedrich Gauss made outstanding contributions to both pure (studied for its own sake) and applied (studied in order to solve specific problems) mathematics.

Gauss's Problem.
A vessel containing 2000 liters of liquid with a concentration 80% alcohol. Every day removed from 15-liter vessel and replaced with another 12 liters of a liquid whose alcohol concentration is only 40%. After how many days the liquid in the vessel reaches 50% ?

Apparently, as set out is a simple problem. This is interesting in terms of resolving them, as was mentioned at the time of Gauss. Solving the problem is not obvious, as will be seen in what follows. From mathematical point of view, solving requires notions and concepts of *higher mathematics in functional equations* (equations with finite differences of order scratchy). The problem was solved by W. LOREY (1935) and A. WALTHER (1936) by two scientific articles. From *the numerical problem* requires specific knowledge of numerical methods to solving equations with finite differences. W. LOREY used a machine to solve the numerical calculation of the difference equations (the solution is obtained after a considerable number of iterations). To make comparison between *algorithmic solutions* obtained computer and *analytical solution* (mathematical method), we present brief time resolve A. Walther.

Solving mathematics (Mathematical method)
We will do the following:

- a *- the quantity of liquid (in liters) contained initially in the vessel;*
- b *- the quantity of liquid that is removed daily from the vessel;*
- c *- the amount of liquid that is added daily vessel;*
- y_0 *- the amount of alcohol per liter (the concentration of alcohol) a liquid vessel at the time of the initially ;*
- y_p *- the amount of alcohol per liter of liquid that is added;*
- y_f *- the amount of alcohol per liter of liquid in the vessel, at the end;*
- x *- number of days (operations fluid replacement);*
- $y(x)$ *- the amount of alcohol per liter of fluid from vessel operations after x replacement fluid.*

Obtain the following functional equation:

$$(a - bx + cx)\, y(x) - (a - bx + c(x-1))\, y(x-1) = c\, y_p$$

General solution is

$$y(x) = y_p + (y_0 - y_p)\frac{\Gamma((a-b)/(b-c))\,\Gamma(a/(b-c)-x)}{\Gamma(a/(b-c))\,\Gamma((a-c)/(b-c)-x)},$$

where $\Gamma(x)$ is Euler's function:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt .$$

In the case $a = 2000$, $b = 15$, $c = 12$, $y_0 = 0.8$, $y_p = 0.4$, $y(x)$ is a polynomial of degree IV:

$$y(x) = 0.4 + 0.4(1-\frac{3x}{1988})(1-\frac{3x}{1991})(1-\frac{3x}{1994})(1-\frac{3x}{1997})$$

By approximation is concluded that the *y (194) = 0.5004515*, *y (195) = 0.4995996*, so after *x = 195* days to get to the concentration $y_f = 0.5$.

Solving using computer (Algorithmics method)

In addition we use the following variables:

z - *the quantity of alcohol in the vessel at a time;*

t - *the quantity of liquid in the vessel at a time;*

y0 - *the concentration of alcohol in the vessel at a time.*

```
algorithm Gauss;
int x;
float a,b,c,y0,yp,yf,z,t;

begin // main
  read a,b,c ;   //liquid quantities
  read y0,yp,yf; //concentrations
  // initializations
  x←1; z←(a-b)*y0+c*yp; t←a-b+c
  while yf < z/t  do
  begin
  x←x+1; y0← z/t; //concentration
  z←(t-b)*y0+c*yp; t←t-b+c;
  end
  write x; // solution
end
```

Execution by the computer program we tested for the following:

a	y0-final	x (days)
2000	0.5004515	195
5000	0.5001438	488
10000	0.5000983	976
100000	0.5000064	9763

Table 1. Solutions of program: some cases

4 Computer Science and Computational Geometry

*"A picture is worth ten thousand as the words" (*Chinese proverb*)*

*"The book of nature is written in the characters of geometry" (*Galileo*)*

The term *Computer Graphics* has several meanings:

- the *digital images* so produced;
- the representation and manipulation of *pictorial data* by a computer;

- the various *technologies* used to create and manipulate such pictorial data;
- the sub-field of computer science, which studies methods for digitally *synthesizing and manipulating visual* content.

The field of computer graphics developed with the emergence of computer graphics hardware. In 1953 has invented the *graphical display* (Graphic Display) and so switched to a new stage in the development and spread of the computer. Possibility of modeling the spatial output (OUTPUT device) could not be achieved by using only bits of memory. Early projects like the Whirlwind (The *Whirlwind computer* was developed at the Massachusetts Institute of Technology; the project's budget was $1 million a year) introduced the CRT (cathode ray tube) as a viable display and interaction interface and introduced the *light pen* as an *input device*. A light pen could be used to draw sketches on the computer using Ivan Sutherland's revolutionary (1963, PhD thesis) Sketchpad software (*Sketchpad* is considered to be the ancestor of modern Computer-Aided Drafting (CAD) programs).

At first graphical representations made on paper using characters (letters and numbers) for images. A plotter is a vector graphics-printing device to print graphical plots that connects to a computer. Graphical representations using character (numeric or alphanumeric) was not a solution to achieve a faithful representation of real objects. The period 1960-1980 after it was invented hardware support; it took research and experiments, models, algorithms and software to use the lighting of a "pixel" (indivisible unit graphics provide a graphical display). Computer displays are made up from small dots called pixels. The word "pixel" was first published in 1965 by Frederic C. Billingsley. Each pixel intensity and colour offering, and their crowd formed a structure of graphic representation (resolution). The intensity of each pixel is variable. This structure is in computer science, which is the calculation in mathematical analysis (Newton, Riemann, Darboux, Leibniz, etc.). System division (discrete process) from the calculation is entirely analogous to the resolution (pixel matrix) provided a graphic display (Vlada 2008; Vlada, Posea, Nistor, Constantinescu, 1992). From that moment was born on Computer Graphics (2D and 3D): drawing a straight segment (Bresenham algorithm), and drawing the circle ellipse, drawing curves and approximation, algorithms for clipping (algorithm Cohen - Sutherland, Hodgman algorithm-Sutherland, Weiler-Atherton algorithm) techniques for 2D and 3D, models of illumination and reflection, raster graphics, vector graphics, texture techniques. Thus were laid the foundations for integrated software solutions and hardware for design, analysis and computer-aided manufacturing (CAD / CAM / CAE). By involving computer use in solving problems in many areas have been defined and solved various requirements and projects in the past were unthinkable.

Road open Computer Graphics was continued for Computational Geometry: polygonal domains, spatial orientation problems and algorithms, triangulation, covering convex 2D and 3D (Quick Hull algorithm, Graham algorithm, the algorithm Jarfis involution, Chan's algorithm), monotonous polygons, Voronoi Diagrams (Fortune algorithm), Delaunay triangulation algorithm, Graph visibility, Dijkstra's algorithm, problems and intersection algorithms, dynamic movement of objects in space, causing the points belonging to a domain (O'Rourke, 1998; Goodman and O'Rourke, 2004).

NOTE: The Jordan Curve Theorem for Polygons.
Any simple closed curve C divides the points of the plane not on C into two distinct domains (with no points in common) of which C is the common boundary. We shall take the case where C is a closed polygon P. The proof given by Camille Jordan (1838 -1922) he was quite complicated and it turned out to be invalid (Hales, 2007). A demonstration of the theorem is given in “*Computational Geometry Student Projects - 1997*” (Toussaint, 1997; CGAL , 2009; Davis, 2006; Goodman and O'Rourke, 2004).

4.1 Green's Theorem and area of a polygon

"*Imagination is more important than knowledge*" Albert Einstein
"The beginning of wisdom is the definition of terms." Socrates

George Green (1793-1841) English mathematician and physicist is known for its contributions through mathematical analysis with applications in the theory of electricity and magnetism. (“*An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism”*, George Green, 1828).

Green's theorem gives the relationship between a *line integral* around a simple closed curve *C* and a *double integral* over the plane region $D \subseteq R^2$ bounded by *C*. In a cartesian system of axes *XOY* is considered domain $D \subseteq R^2$ which has the border curve *C* (*D* be the region bounded by *C*) consists of the meeting closed curves C1, C2, C3, C4 (where C_2 and C_4 are vertical lines).

The curve C_1 is given by parametric equations: $x = x$, $y = g_1(x)$, $a \le x \le b$.
The curve C_3 is given by parametric equations: $x = x$, $y = g_2(x)$, $a \le x \le b$.
It is considered *L* and *M* are functions (class C^1) of (x, y) defined on an *open region* containing *D* and have *continuous partial derivatives*. Define

$$D = \{(x, y) \mid a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

where $g_1(x)$ and $g_2(x)$ are *continuous functions* on $[a,b]$.
Green's formula establishes the relationship between curves integral and double integral.

Green's formula is given by

$$\int_C L\,dx + M\,dy = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dA.$$

In physics, *Green's theorem* is mostly used to solve two-dimensional flow integrals, stating that the sum of fluid outflows at any point inside a volume is equal to the total outflow summed about an enclosing area.

Green's theorem for path of class C^1
Let a plane region $D \subseteq R^2$ bounded by *C*, where $C = FrD = Im\,\gamma$, γ is path of class C^1, $\gamma : [a,b] \to R^2$, $\gamma\,(t) = (x(t), y(t))$, $a \le t \le b$.

Theorem 1.

If a plane region $D \subseteq R^2$ bounded by γ, where γ is path of class C^1 and $FrD = Im\gamma$,

$$m(D) = \frac{1}{2}\oint_{\gamma}(xdy - ydx) \equiv \iint_D dxdy \text{ ,}$$

where *m(D)* is *Jordan measure* (area of *D*).

Proof. Because the assumptions are valid Green theorem, mainly considering *L(x,y)*= - *y/2*, *M(x,y)*= *x/2* and apply Green's theorem.

Green's theorem for polygons

Theorem 2.

If a plane region $D \subseteq R^2$ bounded by γ, where γ is path of class C^1 upon portions and $FrD = Im\gamma$, then

$$\gamma = \gamma_1 \cup \gamma_2 \cup ... \cup \gamma_n \Rightarrow$$

$$m(D) = \frac{1}{2}\sum_{i=1}^{n}\int_0^1 (xdy - ydx) \text{ ,}$$

where γ is path of class C^1 upon portions, and *m(D)* is *Jordan measure* (area of *D*).

Proof.

Let $\gamma : [0,1] \to R^2$, $0 \le t \le 1$ and

$$\begin{cases} \gamma_i : [0,1] \to R^2 \\ \gamma_i(t) = (x(t), y(t)) \end{cases}, 1 \le i \le n$$

Using $\gamma = \gamma_1 \cup \gamma_2 \cup ... \cup \gamma_n \Rightarrow$

$$m(D) = \frac{1}{2}\oint_{\gamma}(xdy - ydx) = \frac{1}{2}\sum_{i=0}^{n}\oint_{\gamma_i}(xdy - ydx).$$

Corolary.

Let the polygon line **P**=$P_1 \ldots P_n$, $P_i(x_i, y_i)$, $1 \le i \le n$, then area of polygon **P** is

$$S = \frac{1}{2}\sum_{i=1}^{n}\begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{vmatrix},$$

where $x_{n+1}=x_1$, $y_{n+1}=y_1$.

Proof.

To see Theorem 2, consider plane region $D \subseteq R^2$ bounded by $\mathbf{P} = FrD$.

Using the bijective application between the real segments [0,1] and [*a*,*b*], given by *φ(t)* = *a* + *t(b-a)*. The polygon line is modeled using the reunion of the γ_i paths parametrically represented as follows:

$$\gamma_i : [0,1] \to R^2, \gamma_i(t) = (x(t), y(t)), 1 \le i \le n,$$

whereas

$$x(t) = x_i + t\,(x_{i+1} - x_i),\ y(t) = y_i + t\,(y_{i+1} - y_i),\ 1 \le i \le n-1$$

noting that for the last path, γ_n parametric equations are

$$x(t) = x_n + t\,(x_1 - x_n),\ y(t) = y_n + t\,(y_1 - y_n).$$

Note. Another formula for the area of any polygon:

$$S = \frac{1}{2}\sum_{i=1}^{n}(x_i + x_{i+1})\cdot(y_{i+1} - y_i)$$ (Davis, 2006; Vlada, 1992)

5 Conclusions

Languages exist therefore, not for communication purposes alone, but particularly for knowledge. Develop programs to solve problems with a computer led to the development and evolution of all sciences. Results and performance obtained through the use of computers have boosted the development of all sciences. Today, information and knowledge are represented differently, shaped and processed. Also, troubleshooting took new dimensions through the use of algorithmic methods. Many issues would have remained unsolved if not using the methods and performance offered by computer. The concepts of language and algorithm were reviewed. Were invented artificial languages used by computer. These languages are not only used to communicate information, but also for processing information and knowledge. Today, all benefit from this science invention. Weight consists of representation and interpretation. Therefore, scientists need to think both in natural environments, but also in virtual environments.

According to the above considerations we conclude with the followingremarks:

1) *Problem solving is based on models of knowledge representation and processing paradigms*;

2) *Processing can be described using some language under a specific interpretation*.

Finally, we propose the following meta-model:

- PROBLEM SOLUTION = MODELLING + PROCESSING
- MODELLING = KNOWLEDGE + REPRESENTATION
- PROCESSING = LANGUAGE + INTERPRETATION

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